## "Multipliers"

## Factors to convert a series

of future payments into a lump
sum

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## Value depends on

- Discount rate
- Payment frequency
- Mortality
- Deferral period
- End of payment period
- ... (complications ...)


## A trivial example

- A one-off payment, made now
- Multiplier = 1 (!)
- Discount rate (ignore)
- Payment frequency = 1
- Mortality (ignore)
- Deferral period = 0
- End of payment period = 0
- ... (complications ...) none
- With no delay in payment, and no uncertainty, the whole value is retained


## A first approximation

- Discount rate - ignore
- Payment frequency e.g. weekly
- Mortality - ignore
- Deferral period - none
- End of payment period e.g. a year
- Value of $\$ 1$ per week for 52 weeks $=52$
- With no discounting and no uncertainty, the multiplier is just the number of payments


## Discounting

- Set by legislation e.g. 5\% pa for Wrongs Act
- Assumed to allow for all effects : tax, inflation ...
- (So we do not allow for inflation in calculations)
- Effectively, a net rate of return pa on money paid to the beneficiary
- Saves arguments about different financial elements
- E.g. roughly 8\% return and 3\% inflation


## Accumulating at 5\%

- $\$ 100$ grows to $\$ 105$ in one year's time
- Which grows to 105 * $1.05=\$ 110.25$ after another year
- Over n years : 100 * 1.05 * $1.05 \ldots \mathrm{n}$ times
- Or \$100 * $1.05^{\wedge} n$ after $n$ years
e.g. $\mathrm{n}=2$ (2 years)

After 2 years, 1.05 * $1.05=1.05^{\wedge} 2=1.1025$

## Discounting at 5\%

- Discounting is the inverse of accumulating
- 90.7 * 1.05 * $1.05=90.7$ * $1.1025=100$

The value now of 100 in 2 years time $=$ $100 / 1.05 / 1.05=100 / 1.05^{\wedge} 2=100 / 1.1025=$ 90.70

- $\left[\$ 100 / 1.05^{\wedge} \mathrm{n}\right]$ grows to $\left[100 / 1.05^{\wedge} \mathrm{n}\right]$ * $1.05^{\wedge} \mathrm{n}$

$$
=\left[100 / 1.05^{\wedge} n\right]^{*} 1.05^{\wedge} n=\$ 100 \text { in } n \text { years }
$$

- So : $\$ 100 / 1.05^{\wedge} n$ is the discounted value of $\$ 100$, for $n$ years, at $5 \%$


## A series of payments: an example

- Discount rate 5\% pa
- \$1 payable every 5 years, for 10 years
- = \$1 now + \$1 in 5 years + \$1 in 10 years
- Value = Value of 1 now
+ value of 1 discounted for 5 years
+ value of 1 discounted for 10 years
- $=1+1 / 1.05^{\wedge} 5+1 / 1.05^{\wedge} 10$
- $=1+0.7835+0.6139=2.3974$


## Weekly multipliers

- Discount rate $5 \%$ pa, $\$ 1$ payable for 3 years
- = \$1 now + \$1 in 1 week + \$1 in 2 weeks ...
- Value = Value of 1 now; + value of 1 discounted for 1 week $+\ldots=145.6$

See tables

## Other frequencies

- Annual, 5 yearly etc are OK. Add deferred values; or Formula is $\left(1-v^{\wedge}\left(n^{*} k\right)\right) /\left(1-v^{\wedge} n\right)$

Where $i=5 \%$

$$
v=1 /(1+i)
$$

n is years between payments
$k$ is number of payments

- E.g. $\$ 30$ each 5 years, for 30 years

Number of payments $=30 / 5+1=7$
$v=1 / 1.05=.9538$
$v^{\wedge} n=v^{\wedge} 5=0.9538^{\wedge} 5=0.7835$
$v^{\wedge}\left(n^{*} k\right)=v^{\wedge}\left(5^{\star} 7\right)=0.9538^{\wedge} 35=0.1813$
Value $=(1-0.1813) /(1-0.7835)=3.7820$

## Allow for delay (s)

- Discount rate - ignore
- Payment frequency e.g. weekly
- Mortality - ignore
- Deferral period - none
- End of payment period e.g. two years
- Value of $\$ 1$ per week for 104 weeks $=104$
- Which is the sum of $\$ 1$ per week for a year, and \$1 per week deferred 1 year


## A general point on deferral

- Deferral will usually reduce the values (would you rather have $\$ 1$ now; or $\$ 1$ in 10 years time?)
- But a series of payments can in general be broken into subsets e.g. $1^{\text {st }}$ year, $2^{\text {nd }}$ year ...
- The multiplier for the whole period may be thought of as the sum of its parts

1 year (immediate)

+ 1 year (deferred 1 year)
+ 1 year (deferred 2 years) ...


## Weekly multipliers

- Discount rate $5 \%$ pa, $\$ 1$ payable for 3 years
- Or: = value of \$1 pw for a year;
+ .. \$1 pw for a year, deferred one year
$+\quad . . \$ 1 \mathrm{pw}$ for a year, deferred two years
- $=50.92+50.92 /[1.05]+50.92 /\left[1.05^{\wedge} 2\right]$
- $=50.92 * 1+50.92 * 0.9524+50.92^{*} 0.9070=145.6$


## Life expectancies

- An average future lifetime
- E.g. a 10 year old, with a life expectancy of 70 has an expectation to live to age 80 (on average)
- Very small chance of early death, reasonable chance of living to at least age 60, small chance of reaching 100


## Mortality rates

- Rates of death at a given age
- Used to calculated life expectancy
- No easy formula

Mortality rates - 7 yo male


## Likelihood of death

- Low chance early
- Low chance very late, as most will have already died
\% dying - 7 yo male



## Mortality

- Shorter life expectancies usually mean smaller values
- Which table?
- ABS annual tables: easier to use, but do not allow for improvements
- ABS projected .. (Zhang case): vary by future year, not just age
- We have been using only the 'with improvement' tables


## Mortality

- Which approach?

Mortality rates

- precise, and more flexible
- Harder to use and understand
- Harder to calculate, and make adjustments to

Life expectancies

- Some loss in precision, usually small
- Much easier to calculate and understand
- Enables deferral tables
- Easy to allow for different life expectancies


## Mortality

- We have been using only the 'life expectancy' approach for about 18 months, and the factors in the law diary are on that basis.
- Assumes survival to assumed date of death, and no payments thereafter


## Life expectancy



## Differences

7 year old male, 2007: Values of \$1 pw

| For life |  | Precise | Approx | Error |
| :---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |
|  | To age 70 | 1035.9 | 1045.7 | $0.9 \%$ |
|  | 60 | 1011.0 | 1020.0 | $0.9 \%$ |
|  | 50 | 982.0 | 988.9 | $0.7 \%$ |
|  | 40 | 833.2 | 938.2 | $0.5 \%$ |
|  | 30 | 719.8 | 855.7 | $0.4 \%$ |
|  | 20 | 502.0 | 721.3 | $0.2 \%$ |
|  | 10 | 145.6 | 502.3 | $0.1 \%$ |
|  |  |  | 145.6 | $0.0 \%$ |

## Complications

- Extreme cases e.g. payments that start beyond life expectancy
- Joint lives
- Payments with offsets e.g. annual repairs to a item purchased every 10 years


## Practical issues

- Errors in calculation e.g. weekly payment * weeks <> payment pa
- Non-contiguous payments - does it matter?
- Annually from now to 31/10/2010
- Annually thereafter
- Lack of precision in dates e.g. paid every 5 years from 2020
- Range estimates e.g. \$2000 to \$3000
- Equating e.g. \$50 each 10 years to $\$ 5$ pa


## Examples

## Vicissitudes

- For any future payment?
- $15 \%$ often used for earnings loss
- No basis?
- What vicissitudes are to be considered?
- If mortality is the only one, and is already allowed for, there may be no need for more


## Checking accuracy

- $5 \%=1 / 20$
- With $\$ 21$, invested at $5 \%$ :
- Pay a dollar
- Leaves \$20
- Which one year later grows to 20*1.05 = 21
- So $\$ 21$ pays $\$ 1$ forever
- So the maximum multiplier pa must be less than 21
- Or $52.18 * 21=1096$ for weekly cases


## Checking accuracy (2)

An alternative method for some cases may be possible, and would provide a check.
e.g. A 5 year weekly payment, deferred 10 years might be calculated either
15 years of weekly payments, of value 555 , minus 10 years of weekly payments (412.9), equals 142.1

This is the same as a 5 year payment, value 231.5 times deferral factor for 10 years $(0.6139)=142.1$

## At the end

- The attendant care cost is usually by far the biggest cost. Is it reasonable?
- Is the weekly value (\$) OK?
- Is the multiplier pa < 21? (5\% case)

Cost < 21 * cost pa

- For short durations of payment, multiplier pa is a bit less than the period of payment
- Are all the items included?
- No duplication? Alternatives allowed for correctly?


## Addedum

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For an expense levied as $0.25 \%$ per annum, the cost of investment management might be estimated as

| Term | Cost $\%$ of |
| :---: | ---: |
| 30 | $2.9 \%$ |
| 50 | $4.1 \%$ |
| 70 | $4.7 \%$ |

